Reg.No. \_\_\_\_\_\_\_\_\_\_\_\_

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**End Semester Examination – Nov/Dec– 2018**

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| **Code :** | **10MA302** | **Duration :** | **3hrs** |
| **Sub. Name :** | **COMMUTATIVE ALGEBRA** | **Max. marks :** | **100** |

**ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)**

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| **Q. No.** | **Sub Div.** | **Questions** | **Marks** |
| 1. | a. | Let  be a finitely generated - module, a multiplicatively closed subset of . Prove that | 7 |
| b. | State and Prove First Uniqueness Theorem. | 7 |
| c. | State and Prove Second Uniqueness Theorem. | 6 |
| (OR) | | | |
| 2. | a. | Let  be an - module. Show that the following are equivalent:  (i)  (ii)  for all prime ideals of ;  (iii)  for all maximal ideals of . | 7 |
| b. | Prove the following  (i) Every ideal in  is an extended ideal.  (ii) If  is an ideal in , then . Hence  if and only if meets  (iii)  is a zero-divisor in  (iv) The prime ideals of  are in one-to-one correspondence  with the prime ideals of  which don’t meet | 10 |
| c. | Let  be a primary ideal in a ring . Show that is the smallest prime ideal containing . | 3 |
| 3. | a. | State and Prove Going-up Theorem. | 10 |
| b. | Show that is a Noetherian -module every submodule of  is finitely generated. | 10 |
| (OR) | | | |
| 4. | a. | State and Prove Going-down Theorem. | 10 |
| b. | Suppose that  has a composition series of length Show that every composition series of has length and every chain in can be extended to a composition series. | 10 |
| 5. | a. | Show that in a Noetherian ring every irreducible ideal is primary. | 8 |
| b. | State and Prove structure theorem for Artin rings. | 12 |
| (OR) | | | |
| 6. | a. | Let  be an ideal in a Noetherian ring. Prove that the prime ideals which belong to are precisely the prime ideals which occur in the set of ideals . | 8 |
| b. | Show that in a Artin ring the nilradical is nilpotent. | 6 |
| c. | Let  be an Artin local ring. Prove that the following are equivalent:  (i) every ideal in  is principal;  (ii) the maximal ideal is principal;  (iii) | 6 |
| 7. | a. | Let  be a Noetherian local domain of dimension one,  its maximal ideal, its residue field. Show that the following are eqiavalent:  (i)  is a discrete valuation ring;  (ii)  is integrally closed;  (iii)  is a principal ideal;  (iv)  (v) Every non-zero ideal is  power of ;  (vi) There exists  such that every non-zero ideal is of the form , | 12 |
| b. | Let  be a local domain. Show that  is a discrete valuation ring every non-zero fractional ideal of  is invertible. | 8 |
| (OR) | | | |
| 8. | a. | Define discrete valuation on a field. | 3 |
| b. | Show that the ring of integers in an algebraic number filed  is a Dedekind domain. | 7 |
| c. | Let  be an integral domain. Prove that  is a Dedekind domain every non-zero fractional ideal of  is invertible. | 10 |
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|  | | **Compulsory:** |  |
| 9. | a. | Show that every ring  has at least one maximal ideal. | 5 |
| b. | Prove that the nilradical of  is the intersection of all the prime ideals of | 10 |
| c. | State and Prove Nakayama’s lemma. | 5 |